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Dynamic Measurements of the Temperature Coefficient of Resistance in the Transient Plane Source Sensor

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Abstract The extended dynamic plane source (EDPS) method is one of the transient methods for measurements of the thermal conductivity and thermal diffusivity in solids. This technique uses a transient plane source (TPS) sensor, which serves as the heat source and thermometer. Its calibration consists of measuring the temperature dependence of the TPS sensor resistance and computing the temperature coefficient of resistance (TCR) using least-squares (LS) estimation. The goal of this study is to calibrate the TPS sensor directly in the apparatus for the EDPS method. The article presents an uncertainty assessment of the TCR measurement. The main sources of uncertainty stem from resistance measurements of the constant resistor and platinum thermometer calibration. The LS estimate of the TCR in a nickel TPS sensor is $4.83 \times 10^{-3}\,\mathrm{K}^{-1}$ at 20 °C and $4.57 \times 10^{-3}\,\mathrm{K}^{-1}$ at 45 °C with a combined standard uncertainty better than $0.04 \times 10^{-3}\,\mathrm{K}^{-1}$, which is 0.7 %.

Keywords Plane source sensor · Temperature coefficient of resistance · Thermophysical parameters · Transient method · Uncertainty

1 Introduction

Transient methods [1] for measurements of thermophysical parameters (thermal conductivity λ and thermal diffusivity a) are based on generation of a dynamic

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temperature field inside a specimen. The theoretical model is characterized by a temperature function, which is a solution of the heat equation with boundary and initial conditions corresponding to the experimental arrangement. The principle of the evaluation consists of fitting the temperature function to the experimental points (temperature response), determined from the measurements of the transient plane source (TPS) sensor resistance. Hence, the calibration of the sensor is necessary for obtaining reliable values of the thermal conductivity. The following transient methods use the plane heat source, which simultaneously serves as the thermometer: TPS method [2], dynamic plane source (DPS) method [3], and extended dynamic plane source (EDPS) method [4–6].

The measurement of the temperature coefficient of resistance (TCR) in the TPS sensor, made from nickel foil in the form of a bifilar spiral, was described in [2]. The temperature dependence of the TPS sensor resistance was measured in the range from 35 K to 300 K using a Pt thermometer. The experimental data were fitted to a fourth-order polynomial to get an analytical form, so the TCR could be obtained from

$$\alpha(T) = \frac{1}{R(T)} \frac{\mathrm{d}R(T)}{\mathrm{d}T}. \tag{1}$$

However, experimental details, error analysis, and the uncertainty of the results are missing in this study.

To get reliable results of the thermal-conductivity measurement, the particular TPS sensor should be calibrated or at least one of the series made from the same material. The aim of this study is to design the apparatus for TPS sensor calibration as a modification of the EDPS experimental equipment. The uncertainty assessment will be performed according to GUM [7]. The Type A evaluation of uncertainty will be done by means of the residual variance. In the Type B evaluation the manufacturer's declared errors are considered as the half-width a of the symmetrical rectangular distribution and the standard uncertainty of the input quantity is calculated as

$$u_{\mathrm{B}}(x) = a/\sqrt{3}. \tag{2}$$

2 Extended Dynamic Plane Source Method

The EDPS method is arranged for one-dimensional heat flow into a finite solid with a low thermal conductivity. Figure 1 shows the TPS sensor in the form of a meander, made from a 20 μm thick nickel foil and covered on both sides with a 25 μm kapton layer. The sensor, 30 mm in diameter, is placed between two identical specimens with the same cross section. The heat sink, made of a very good heat conduction material (aluminum), provides isothermal boundary conditions for the experiment. Heat is produced by the passage of an electrical current in the form of a step-wise function through the TPS sensor. The temperature response is determined by measuring the time dependence of the TPS sensor resistance.

The apparatus enables one to increase the temperature of the experiment. It consists of a platinum thermometer (Pt100), two heating elements, a multichannel PC plug-in

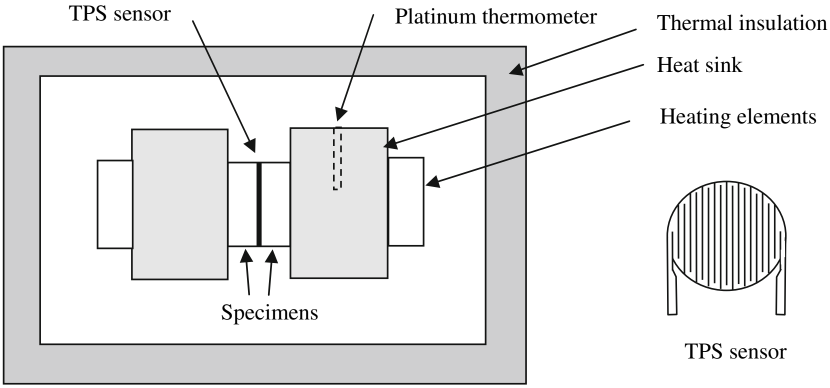


Fig. 1 Arrangement of the experiment

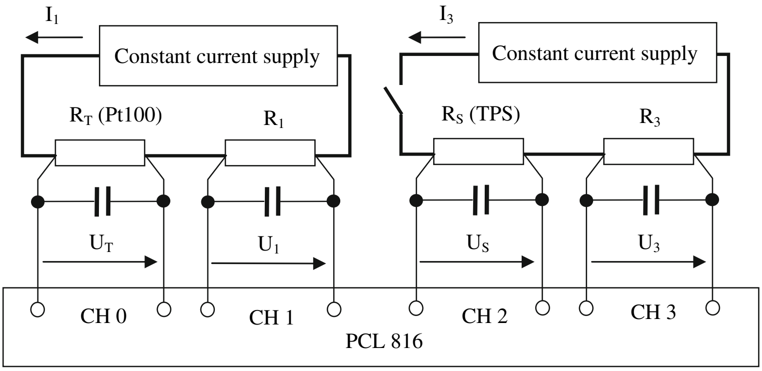


Fig. 2 Experimental circuit design

card (PCL-816), and a power DA converter. A proportional integral (PI) controller [8] realized by PC software is used for temperature control. It is based on a periodic measurement of the temperature and computation of the manipulated variable (heating power). PI control allows good temperature homogeneity in the heat sink and specimens and a stability better than ±5 mK.

The resistances of the platinum thermometer and TPS sensor are calculated by

$$R_{\mathrm{T}} = R_1 \frac{U_{\mathrm{T}}}{U_1} \quad R_{\mathrm{S}} = R_3 \frac{U_{\mathrm{S}}}{U_3} \tag{3}$$

where R_1 and R_3 are the constant resistors. The voltages are measured as shown in Fig. 2. Currents in the circuits were set to $I_1 = 1.2\,\mathrm{mA}$ and $I_3 = 300\,\mathrm{mA}$. The temperature of the Pt100 was determined using the following formula [9]:

$$R_{\mathrm{T}} = R_0 \left(1 + \alpha_{\mathrm{T}} T + \beta_{\mathrm{T}} T^2\right), \tag{4}$$

where $R_0 = 100.00\,\Omega$, $\alpha_{\mathrm{T}} = 3.9092 \times 10^{-3}\,\mathrm{K}^{-1}$, and $\beta_{\mathrm{T}} = -5.917 \times 10^{-7}\,\mathrm{K}^{-2}$.

3 Temperature Coefficient of Resistance Measurement

In order to use the described apparatus for TCR measurements, the specimens in Fig. 1 were removed. The TPS sensor was coated with silicon oil to improve thermal contact and clamped between two aluminum blocks (heat sink). The temperature of the heat sink and TPS sensor was stabilized for about an hour. The temperature was measured in the steady state, then the current I_3 was switched on and the transient response of the TPS sensor resistance was measured, as shown in Fig. 3. The first sample value r_1 (0.1 s after switching) was taken as the TPS sensor resistance R_S at the measured temperature T.

The measured data were fitted to the following polynomial [10]:

$$R(T) = a_0 + a_1 T + \cdots + a_k T^k. \tag{5}$$

The model of the measurement in matrix notation is given by the form,

$$\mathbf{R}_\mathrm{S} - \boldsymbol{\epsilon} = \mathbf{X} \cdot \mathbf{a} \tag{6}$$

where $\mathbf{R}_\mathrm{S}$ is the observation vector of the TPS sensor resistance measured at 6 points (T_i) in the interval from 20 °C to 45 °C, $\mathbf{a}$ is the vector of unknown parameters, $\boldsymbol{\epsilon}$ is the vector of errors, and $\mathbf{X}$ is the sensitivity matrix defined by

$$\{\mathbf{X}\}_{ij} = T_i^j \tag{7}$$

The least-squares (LS) estimate of the parameter vector is

$$\hat{\mathbf{a}} = \mathbf{M} \cdot \mathbf{X}^T \cdot \mathbf{R}_\mathrm{S} \tag{8}$$

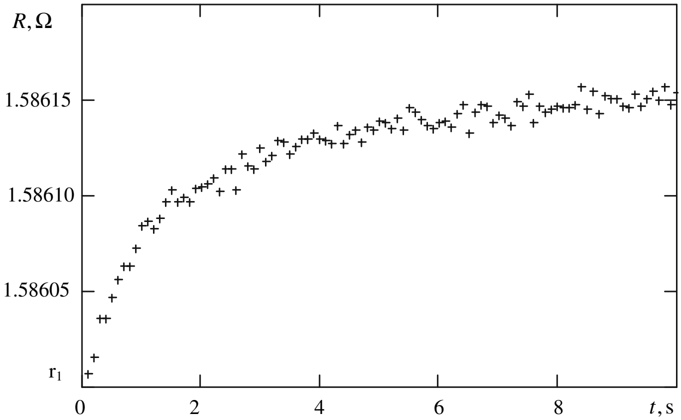


Fig. 3 Time dependence of the TPS sensor resistance after switching on the current I_3

Table 1 Results of LS estimation for the degree of polynomial $k = 2$

T (°C)	$\alpha \times 10^3 (\mathrm{K}^{-1})$	$\mathrm{d}R/\mathrm{d}T \times 10^3 (\Omega \cdot \mathrm{K}^{-1})$
20	4.828	6.81
45	4.568	7.16

α is the TCR of TPS sensor and $\mathrm{d}R/\mathrm{d}T$ is the slope of $R(T)$ dependence

where the matrix **M** is given by

$$\mathbf{M} = (\mathbf{X}^T \cdot \mathbf{X})^{-1} \tag{9}$$

We tested the significance of the model parameters using the test statistic [11],

$$t = \frac{a_j}{s\sqrt{\{\mathbf{M}\}_{jj}}} \tag{10}$$

which has a Student's t distribution if the null hypothesis (H_0: $a_j = 0$) is true and s is the standard deviation of residuals. The test showed that parameters a_3, a_4, and a_5 are not statistically significant at a level of 0.05; hence, the second-order polynomial describes the conditions acceptably and will be used in the uncertainty evaluation.

Once we have the parameter estimates, the TCR of the TPS sensor can be computed using Eqs. 1 and 5. The results for two temperatures are presented in Table 1. As the slope of the temperature dependence of the TPS sensor resistance changes only 5 % in the temperature interval of interest, the dependence will be considered as linear and the uncertainty of the temperature measurement $u(T)$ can be propagated into the uncertainty of the TPS sensor resistance measurement as follows:

$$u(R) = u(T)\,\frac{\mathrm{d}R}{\mathrm{d}T} \approx 0.007u(T)\,\Omega \cdot \mathrm{K}^{-1} \tag{11}$$

4 Measurement Uncertainty of Input Quantities

Both constant resistors, $R_1 = 136.40\,\Omega$ and $R_3 = 1.009\,\Omega$, were measured by a 6½ digit multimeter with uncertainties determined according to manufacturer's specifications as $u_\mathrm{m}(R_1) = 20\,\mathrm{m}\Omega$ and $u_\mathrm{m}(R_3) = 4.5\,\mathrm{m}\Omega$. The uncertainty contribution due to the temperature instability of the resistor R can be estimated as

$$u_\mathrm{t}(R) = R\,\alpha_\mathrm{R}(T_\mathrm{max} - T_\mathrm{min})/6 \tag{12}$$

where the TCR of the resistor is within $\pm\alpha_R = 25\,\mathrm{ppm} \cdot \mathrm{K}^{-1}$ and the laboratory temperature is assumed to be in the interval from $T_\mathrm{min} = 17\,°\mathrm{C}$ to $T_\mathrm{max} = 33\,°\mathrm{C}$. Both uncertainties were combined by root-sum-square addition giving the following results: $u_\mathrm{B}(R_1) = 22\ \mathrm{m}\Omega$ and $u_\mathrm{B}(R_3) = 4.5\ \mathrm{m}\Omega$.

Voltages in the circuit were measured by a 16-bit resolution card PCL-816. The gain error is reported as 0.04 % of full scale range (FSR), and a maximum drift of $U_D = 40\,\mu V$ is observed. To suppress quantization and electrical noise, voltages U_1, U_T, and U_3, U_S are sampled and averaged 4,000 and 1,500 times per channel over the period of 1 s and 60 ms, respectively. The connection in Fig. 2 also suppresses the gain error as the voltages are strongly correlated. Hence, the standard uncertainty of the voltage measurement can be reasonably estimated as $u_B(U) = 40\,\mu V/\sqrt{3} = 23\,\mu V$ with correlation coefficients $r(U_1, U_T) = r(U_3, U_S) = 1$. Using the law of propagation of uncertainty [7], it can be shown that the contribution from the voltage measurement can be neglected and the uncertainty of the measurement of the Pt100 and TPS sensor resistance can be estimated as

$$u_B(R_T) = \frac{R_T}{R_1} u_B(R_1) = 18\,\text{m}\Omega, \quad u_B(R_S) = \frac{R_S}{R_3} u_B(R_3) = 6.7\,\text{m}\Omega \tag{13}$$

The uncertainty of the Pt100 resistance measurement produces an uncertainty in the temperature measurement,

$$u_B(T) = \frac{\partial T}{\partial R_T} u_B(R_T) \approx \frac{u_B(R_T)}{R_0 \alpha_T} = 46\,\text{mK} \tag{14}$$

which can be transformed using Eq. 11 to $u'_B(R_S) = 0.3\,\text{m}\Omega$ and neglected when compared to Eq. 13.

As the current I_3 causes self-heating of the TPS sensor, the value of the first sample r_1 (Fig. 3) may be different from the value r_0 corresponding to the measured value of the temperature. This can cause a systematic error in the measurement of the TPS sensor resistance. This effect was analyzed by calculating the temperature increase in an adiabatic state $\Delta T = \Delta t(P/C)$ over the time $\Delta t = 0.1$ s, where P is the input power and C is the heat capacity of the sensor. The transformation using Eq. 11 demonstrated that the associated uncertainty of the TPS sensor resistance measurement is 1.1 mΩ, which can be ignored when compared to Eq. 13.

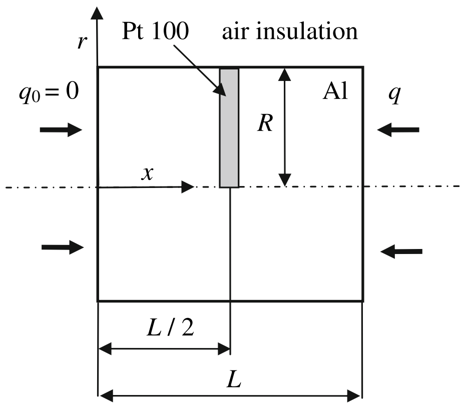
The self-heating effect of the platinum thermometer was investigated using a two-current method [12], with the result that the systematic error of the temperature measurement is about 10 mK, which can be neglected.

5 Temperature Uniformity in the Heat Sink

In the steady state, the temperature measured in one location (Pt100) may be different from the temperature of the TPS sensor. This may cause a systematic error of the temperature measurement. To analyze this effect, we decided to create a simple model of the temperature field in the heat sink, as illustrated in Fig. 4.

(i) The heating element is placed at $x = L$ and provides a constant heat current density q.
(ii) The TPS sensor is placed at $x = 0$ and the thermometer at $x = L/2$.
(iii) There is no heat flow into the TPS sensor, $q_0 = 0$.

Fig. 4 Analysis of the temperature field in the heat sink



(iv) The heat is dissipated through the cylinder surface into the surrounding air with a constant temperature.

The heat equation in cylindrical coordinates is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \tag{15}$$

where T is the difference between the temperature in the place $[x, r]$ and the temperature of the surrounding air. The boundary conditions are

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \tag{16}$$

$$-\lambda \left.\frac{\partial T}{\partial x}\right|_{x=L} = -q \tag{17}$$

$$-\lambda \left.\frac{\partial T}{\partial r}\right|_{r=R} = hT(x, R) \tag{18}$$

Using the finite Hankel transform [13], the temperature field in the heat sink can be expressed by the formula,

$$T(x, r) = 2\,T_0 \sum_{\xi} \frac{\beta}{\xi(\xi^2 + \beta^2)} \frac{J_0\left(\xi \frac{r}{R}\right)}{J_0(\xi)} \frac{\cosh\left(\xi \frac{x}{R}\right)}{\sinh\left(\xi \frac{L}{R}\right)} \tag{19}$$

where ξ's are the roots of the following equation:

$$\beta J_0(\xi) - \xi J_1(\xi) = 0 \tag{20}$$

$\beta = Rh/\lambda$, $T_0 = qR/\lambda$, h is the heat-transfer coefficient, and λ is the thermal conductivity of the heat sink (aluminum). Figure 5 shows the temperature profiles computed for the worst-case situation, i.e., $L = R = 0.03\,\text{m}$, $h = 10\,\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$,

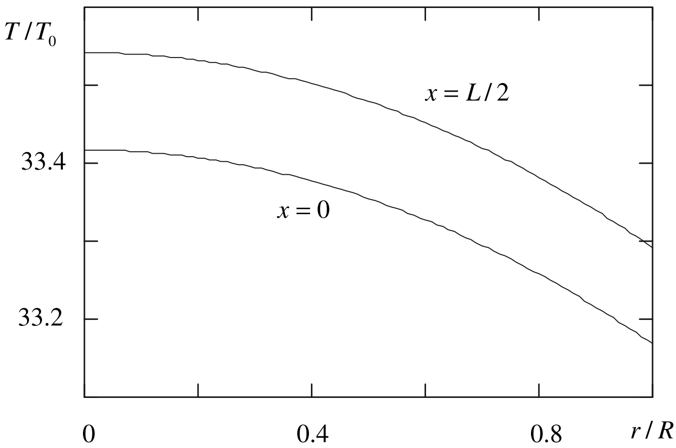


Fig. 5 Temperature profiles in the place of TPS sensor ($x = 0$) and thermometer ($x = L/2$)

$\lambda = 200\,\mathrm{W \cdot m^{-1} \cdot K^{-1}}$, and $q = 210\,\mathrm{W \cdot m^{-2}}$. The analysis demonstrated that the temperature difference between the thermometer and the TPS sensor will be less than $0.4\,T_0$, i.e., 10 mK, which can be ignored in the uncertainty assessment.

6 Uncertainty Assessment

The thermometer Pt100 has been calibrated by the manufacturer with a declared standard uncertainty $u(T) = 0.1\,\mathrm{K}$. Considering the worst-case situation, we must assume that the error is not constant for all temperatures of the experiment. We assume that ε_1 and ε_2 are independent errors at temperatures $T_1 = 20\,°\mathrm{C}$ and $T_2 = 45\,°\mathrm{C}$, respectively. Then the errors for other temperatures in the interval are given by the linear form $f(T) = e_0 + e_1 T$, where

$$e_0 = \frac{\varepsilon_1 T_2 - \varepsilon_2 T_1}{T_2 - T_1} \quad e_1 = \frac{\varepsilon_2 - \varepsilon_1}{T_2 - T_1} \tag{21}$$

Using the law of uncertainty propagation, we have

$$u_\mathrm{B}(e_0) = \frac{\sqrt{T_1^2 + T_2^2}}{T_2 - T_1} u(\varepsilon) = 1.4\,\mathrm{m\Omega} \quad u_\mathrm{B}(e_1) = \frac{\sqrt{2}}{T_2 - T_1} u(\varepsilon) = 0.04\,\mathrm{m\Omega \cdot K^{-1}} \tag{22}$$

where the uncertainty of the temperature measurement $u(T)$ was transformed to the uncertainty of the estimate of the TPS sensor resistance $u(\varepsilon)$ using Eq. 11. Then the vector of errors in Eq. 6 can be expressed as

$$\boldsymbol{\epsilon} = \varepsilon_\mathrm{S} + e_0 + e_1\,\mathbf{T} \tag{23}$$

where ε_S is the error of the measurement of the TPS sensor resistance and $\mathbf{T}$ is the vector of the experimental temperatures. The covariance matrix [14] of the parameter vector will be

$$\mathbf{U}_{\hat{\mathbf{a}}} = \left(\mathbf{X}^T \mathbf{X}\right)^{-1} u_A^2(R_S) + \mathbf{Q}^T \mathbf{Q}\, u_B^2(R_S) + \mathbf{W}^T \mathbf{W}\, u_B^2(e_1) \tag{24}$$

where $\mathbf{Q} = (1, 0, 0), \mathbf{W} = (0, 1, 0), u_A(R_S) = 0.02\,\mathrm{m}\Omega$ is the Type A standard uncertainty given by the standard deviation of residuals and $u_B(e_0)$ has been neglected compared to $u_B(R_S)$.

Finally, using the law of uncertainty propagation in matrix notation, the standard uncertainty of the TCR measurement in the TPS sensor will be

$$u(\alpha) = \sqrt{\mathbf{c}^T\, \mathbf{U}_{\hat{\mathbf{a}}}\, \mathbf{c}} \tag{25}$$

where $\mathbf{c}$ is the vector of sensitivity coefficients [7] defined by

$$c_j = \frac{\partial \alpha}{\partial a_j} \tag{26}$$

and α is given by Eqs. 1 and 5.

The augend in Eq. 24, designated as the LS component, characterizes the measured point dispersion produced by random errors. This effect can be caused by electrical noise, variations in temperature, and other unknown variations in the time scale of the measurement. The second addend represents the errors which are constant for all measured points, and the last term corresponds to errors which are proportional to the temperature.

Alternatively, we can evaluate the contribution to the uncertainty of the TCR measurement for each input source separately and combine the results by root-sum-square addition. Table 2 presents the results of the uncertainty analysis, where the significance of each source is clearly seen. Figure 6 presents the results of the TCR in the TPS sensor measurement as a function of the experimental temperature, which can be safely approximated by a linear dependence,

$$\alpha(T) = 5.03 \times 10^{-3}\,\mathrm{K}^{-1} - 0.0104 \times 10^{-3}\,\mathrm{K}^{-2} \tag{27}$$

with all residuals less than $0.002 \times 10^{-3}\,\mathrm{K}^{-1}$. The figure also shows the band about the estimated values that encompasses 95 % of the probability distribution defined by [7]

$$\alpha \pm U(\alpha) = \alpha \pm 2u(\alpha) \tag{28}$$

where $U(\alpha)$ is the expanded uncertainty of the TCR estimation.

Table 2 Uncertainty budget for measurement of TCR in TPS sensor at two temperatures

Source of uncertainty	Standard uncertainty	Value	Standard uncertainty	
			$u(\alpha_{20^\circ C}) \times 10^3$ (K^{-1})	$u(\alpha_{45^\circ C}) \times 10^3$ (K^{-1})
LS component	$u_A(R_S)$	0.02 mΩ	0.003	0.002
R_1 measurement	$u_m(R_1)$	20 mΩ	0.001	0.001
Temperature instability	$u_t(R_1)$	9 mΩ	–	–
R_3 measurement	$u_m(R_3)$	4.5 mΩ	0.023	0.019
Temperature instability	$u_t(R_3)$	0.1 mΩ	–	–
Voltage measurement	$u_B(U)$	0.02 mV	–	–
Temperature uniformity	$u_U(T)$	0.01 K	–	–
Pt100 self-heating	$u_T(T)$	0.01 K	–	–
TPS sensor self-heating	$u_S(T)$	0.15 K	0.004	0.003
Pt100 calibration	$u(T)$	0.1 K	0.027	0.021
Combined uncertainty			0.036	0.029

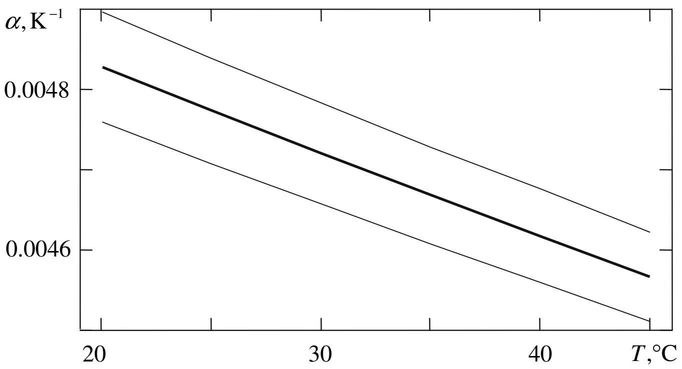


Fig. 6 95 % band of TCR measurement results

7 Conclusions

The article presents a practical method for the TPS sensor calibration, which employs the apparatus for EDPS measurements. The standard oil bath is replaced by electronic temperature control in a solid-state thermostat. The calibration consists of measuring the temperature dependence of the TPS sensor resistance and computing the TCR using a LS estimation. So the user of the EDPS method can easily verify the TPS sensor to be sure that his results are reliable.

This can only be achieved if the measurement results will be accompanied by a quantitative assessment of its uncertainty. The main sources of uncertainty stem from the measurement of the constant resistor R_3 resistance and from the calibration of the platinum thermometer Pt100. The contribution of all other sources of uncertainty can be ignored, as shown in Table 2.

The LS estimate of the TCR in a nickel TPS sensor becomes $4.83 \times 10^{-3}\,\mathrm{K}^{-1}$ at $20\,^{\circ}\mathrm{C}$ and $4.57 \times 10^{-3}\,\mathrm{K}^{-1}$ at $45\,^{\circ}\mathrm{C}$. The measured values in this temperature interval can be approximated by a linear function of temperature. The uncertainty analysis demonstrated that the combined standard uncertainty of the TCR estimation does not exceed $0.04 \times 10^{-3}\,\mathrm{K}^{-1}$, which is 0.7 %. This value is sufficient, because it will be the contribution to the uncertainty of the thermal-conductivity measurement by the EDPS method.

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